
IMPROVED METHODS OF INFERENCE IN ECONOMETRICS

GEORGE G. JUDGE

*UNIVERSITY OF ILLINOIS
UNIVERSITY OF CALIFORNIA, BERKELEY*

and

THOMAS A. YANCEY

UNIVERSITY OF ILLINOIS



1986

NORTH-HOLLAND
AMSTERDAM • NEW YORK • OXFORD • TOKYO

CONTENTS

1. Introduction	1
1.1. Organization of the Book	3
1.2. References	4

Part I – The Inferential and Decision Framework

2. The Measure of Performance	7
2.1. The Information Space	7
2.2. The Classical Approach	7
2.3. The Decision Theory Approach	8
2.3.1. Choosing a decision rule	9
2.3.2. The measure space	11
2.4. References	13
3. Some Alternative Statistical Models, Estimators and Tests	15
3.1. The Classical Linear Statistical Model, Estimator and Test Statistic	15
3.2. Measures of Performance	17
3.3. Some Alternative Estimators	20
3.3.1. Exact non-sample information	20
3.3.2. Stochastic non-sample information	22
3.3.3. An informative proper prior distribution	24
3.4. Pre-Test Estimators	25
3.4.1. Conventional equality pre-test estimator	26
3.5. Conventional Estimator Inadmissibility and the Stein-Rule Alternatives	28
3.5.1. Estimation under squared error loss	29
3.5.2. Stein-like rules under weighted squared error loss	34
3.6. Estimation with Non-Normal Errors	37
3.7. Some Final Comments	38
3.8. References	39

Part II – Inequality Estimation and Hypothesis Testing

4. Inequality Estimation and Hypothesis Testing of the Location Parameter of a Normal Random Variable	43
4.1. Introduction	43

4.2. The Inequality Estimator and Its Sampling Properties	43
4.2.1. The mean of g^*	45
4.2.2. The risk of the inequality estimator	46
4.2.3. Variance of the inequality restricted estimator	49
4.2.4. Comment	51
4.2.5. Extensions	52
4.2.6. Summary remarks	55
4.3. Inequality Hypothesis Testing	56
4.3.1. A hypothesis testing framework	56
4.3.2. Test statistic and pre-test estimator	57
4.3.3. Mean of the inequality pre-test estimator	58
4.3.4. Risk of the inequality pre-test estimator	60
4.3.5. Variance of the pre-test estimator	63
4.3.6. Hypothesis testing, σ^2 unknown	63
4.3.7. Concluding remarks	66
4.4. The Sampling Performance of Pre-Test Estimators of the Scale Parameter Under Squared Error Loss	66
4.4.1. The statistical model and estimators	66
4.4.2. The pre-test estimator $\hat{\sigma}^2$	67
4.4.3. The inequality pre-test estimator $\hat{\sigma}^{\dagger 2}$	68
4.4.4. Minimum mean square pre-test estimator $\hat{\sigma}^{\hat{2}}$	70
4.4.5. Some final remarks	71
4.5. References	71
Appendix 4.A: A Theorem and Some Corollaries	72
Appendix 4.B: Derivative of the Bias of the Inequality Restricted Estimator g^*	76
Appendix 4.C: Convergence Theorem	77
Appendix 4.D: Estimator Bias and Risk Values	78
5. General Linear Statistical Model and a Single Linear Inequality Constraint	81
5.1. Introduction	81
5.2. The Statistical Model and Estimators	81
5.3. The Mean of the Inequality Restricted Estimator	85
5.4. The Risk Function of the Inequality Restricted Estimator	87
5.4.1. The measure space	87
5.4.2. The risk	88
5.5. Mean and Covariance of b^*	91
5.6. Generalizations	93
5.7. Hypothesis Testing	94
5.7.1. Test statistic and pre-test estimator, σ^2 known	95
5.7.2. Pre-test mean	96
5.7.3. The risk	97

5.8. Some Comments	98
5.9. References	99
6. Inequality Restricted Estimation of Two or More Location Parameters: The Orthonormal Case	101
6.1. The Inequality Restricted Estimator with Two Restrictions	101
6.1.1. The mean and bias of the IRMLE	103
6.1.2. The risk function of the inequality restricted estimator	106
6.1.3. The variance-covariance matrix	109
6.2. The IRMLE with Three Restrictions	109
6.2.1. The mean and bias	112
6.2.2. The risk function and variances	113
6.3. The IRMLE with K Restrictions	114
6.3.1. The risk function	115
6.3.2. The variance-covariance matrix	116
6.4. Summary and Conclusions	117
6.5. References	117
7. An Inequality Restricted Stein Rule Estimator	119
7.1. Statistical Model and Estimators	119
7.2. The Inequality Prior Information Estimator and Its Risk	120
7.3. An Inequality Restricted Stein Estimator (IRSE)	122
7.3.1. The estimator	122
7.3.2. Risk of the Stein inequality estimator	123
7.4. Estimator Risk Comparisons	125
7.5. Risk Evaluations	127
7.6. Bias, Variance and Component Risks	129
7.7. Positive Part Stein Estimation, Restricted and Unrestricted	131
7.8. Summary and Conclusions	133
7.9. References	134
Appendix	134
8. Hypothesis Testing for Two or More Inequalities: The Orthogonal Case	137
8.1. Hypothesis Testing with Equality Constraints	138
8.2. Hypothesis Testing with Two or More Inequality Constraints	139
8.2.1. Hypothesis testing with two inequality constraints	139
8.2.2. A likelihood ratio test: Known variance case, $H_0: \theta \geq 0$	142
8.2.3. A likelihood ratio test: Variance unknown, $H_0: \theta = 0$	144
8.2.4. Hypothesis testing with three inequality constraints:	
Known variance case	145
8.2.5. Hypothesis testing with K inequality constraints	149

8.2.6. Illustrative c^2 values for alternative α levels and numbers of inequalities	151
8.3. Power Function Comparisons: Two Inequality Hypotheses	152
8.3.1. The known variance case	152
8.3.2. The unknown variance case	157
8.4. Hypothesis Testing with an Unknown Variance	158
8.4.1. The two inequality hypothesis case	159
8.4.2. Hypothesis testing with K constraints	164
8.5. Mixed Inequality and Equality Hypotheses	165
8.5.1. The one equality and one inequality hypothesis case: Variance known	165
8.5.2. J inequalities and $(H - J)$ equality hypotheses	166
8.6. One-Sided Confidence Regions	171
8.7. Summary and Conclusions	173
8.8. References	174
Appendix	174
9. Inequality Restricted Estimation: General Design and Restriction Matrices	179
9.1. The General Inequality Restricted Estimator	179
9.1.1. Kuhn-Tucker conditions and a primal-dual formulation	180
9.1.2. Expressions for the inequality restricted estimator	183
9.2. The Reparameterized Model	186
9.2.1. The inequality restricted estimator	189
9.3. The Mean of the Inequality Restricted Estimator	191
9.4. The Risk Function of the Inequality Restricted Estimator	193
9.4.1. Measure space	193
9.4.2. The risk	193
9.5. Risk Matrix for b^*	199
9.6. Mean and Covariance of b^*	204
9.7. Summary	205
9.8. References	205
10. Inequality Hypothesis Testing: General Case	207
10.1. Introduction	207
10.2. Inequality Hypothesis Testing: General R and X for Two Parameters	207
10.2.1. Hypothesis testing in non-orthogonal cases	208
10.2.2. Testing general linear inequality hypotheses	210
10.3. Hypothesis Testing for the Linear Model in the General X and R Case	216
10.4. Inequality Hypothesis Testing: Unknown Variance	218
10.4.1. $H_0: A\theta = 0$ and $H_a: A\theta \geq 0$	219
10.4.2. $H_0: A\theta \geq 0$ and $H_a: A\theta \not\geq 0$	222

10.5. Confidence Regions with General X in Two or More Dimensions	225
10.6. Summary and Conclusions	228
10.7. References	229
Appendix	230

Part III – Some Sampling Results for the Stein Family of Estimators

11. Assessing the Precision of Stein's Estimator	239
11.1. The Statistical Model and Estimators	239
11.2. The Empirical Bayes Estimator	240
11.2.1. Stein empirical Bayes confidence interval	241
11.3. The Bootstrap Approach	243
11.4. The Sampling Experiment Results	243
11.5. Summary and Implications	245
11.6. References	246
12. Some Evaluations of the Sampling Performance of the Limited Translation and New Stein Estimators	247
12.1. Introduction	247
12.2. The Limited Translation Estimator	248
12.2.1. Sampling characteristics of the limited translation estimator	249
12.2.2. Conclusions	250
12.3. The New Stein Estimator	251
12.3.1. Sampling characteristics of the new Stein estimator	254
12.4. The Use of the New Stein J^* as a Criterion in Model Selection	255
12.4.1. A Monte Carlo sampling experiment	256
12.5. References	258
13. Estimation Under Non-Normal Errors and Quadratic Loss	259
13.1. Introduction	259
13.2. The Statistical Model and Estimators	260
13.2.1. Maximum likelihood estimators	260
13.2.2. The L estimators	261
13.2.3. James and Stein estimators	262
13.2.4. New Stein estimators	263
13.3. The Sampling Experiment	265
13.3.1. Independent univariate Student- t	265
13.3.2. Multivariate Student- t	266
13.3.3. The new Stein estimator	267
13.3.4. Component risks	269

13.4. Concluding Remarks	271
13.5. References	271
14. Estimation and Hypothesis Testing in the Case of Possible Heteroskedasticity	273
14.1. Statistical Model and Traditional Estimators	273
14.2. Improved Estimators	275
14.2.1. Known scale parameters (least squares format)	276
14.2.2. Known scale parameters (Aitken format)	277
14.2.3. Unknown scale parameters (least squares format)	278
14.2.4. Unknown scale parameters (Aitken format)	281
14.3. Empirical Risk Functions	283
14.4. Non-Optimality of the Traditional Pre-Test Estimator	285
14.5. Summary	287
14.6. References	288
Subject Index	289