

# **Dynamic Copula Methods in Finance**

**Umberto Cherubini  
Fabio Gobbi  
Sabrina Mulinacci  
Silvia Romagnoli**



A John Wiley & Sons, Ltd., Publication

# Contents

<b>Preface</b>	<b>ix</b>
<b>1 Correlation Risk in Finance</b>	<b>1</b>
1.1 Correlation Risk in Pricing and Risk Management	1
1.2 Implied vs Realized Correlation	3
1.3 Bottom-up vs Top-down Models	4
1.4 Copula Functions	4
1.5 Spatial and Temporal Dependence	5
1.6 Long-range Dependence	5
1.7 Multivariate GARCH Models	7
1.8 Copulas and Convolution	8
<b>2 Copula Functions: The State of the Art</b>	<b>11</b>
2.1 Copula Functions: The Basic Recipe	11
2.2 Market Co-movements	14
2.3 Delta Hedging Multivariate Digital Products	16
2.4 Linear Correlation	19
2.5 Rank Correlation	20
2.6 Multivariate Spearman's Rho	22
2.7 Survival Copulas and Radial Symmetry	23
2.8 Copula Volume and Survival Copulas	24
2.9 Tail Dependence	27
2.10 Long/Short Correlation	27
2.11 Families of Copulas	29
2.11.1 Elliptical Copulas	29
2.11.2 Archimedean Copulas	31
2.12 Kendall Function	33
2.13 Exchangeability	34
2.14 Hierarchical Copulas	35
2.15 Conditional Probability and Factor Copulas	39
2.16 Copula Density and Vine Copulas	42
2.17 Dynamic Copulas	45
2.17.1 Conditional Copulas	45
2.17.2 Pseudo-copulas	46

<b>3 Copula Functions and Asset Price Dynamics</b>	<b>49</b>
3.1 The Dynamics of Speculative Prices	49
3.2 Copulas and Markov Processes: The DNO approach	51
3.2.1 The $*$ and $\star$ Product Operators	52
3.2.2 Product Operators and Markov Processes	55
3.2.3 Self-similar Copulas	58
3.2.4 Simulating Markov Chains with Copulas	62
3.3 Time-changed Brownian Copulas	63
3.3.1 CEV Clock Brownian Copulas	64
3.3.2 VG Clock Brownian Copulas	65
3.4 Copulas and Martingale Processes	66
3.4.1 $C$ -Convolution	67
3.4.2 Markov Processes with Independent Increments	75
3.4.3 Markov Processes with Dependent Increments	78
3.4.4 Extracting Dependent Increments in Markov Processes	81
3.4.5 Martingale Processes	83
3.5 Multivariate Processes	86
3.5.1 Multivariate Markov Processes	86
3.5.2 Granger Causality and the Martingale Condition	88
<b>4 Copula-based Econometrics of Dynamic Processes</b>	<b>91</b>
4.1 Dynamic Copula Quantile Regressions	91
4.2 Copula-based Markov Processes: Non-linear Quantile Autoregression	93
4.3 Copula-based Markov Processes: Semi-parametric Estimation	99
4.4 Copula-based Markov Processes: Non-parametric Estimation	108
4.5 Copula-based Markov Processes: Mixing Properties	110
4.6 Persistence and Long Memory	113
4.7 $C$ -convolution-based Markov Processes: The Likelihood Function	116
<b>5 Multivariate Equity Products</b>	<b>121</b>
5.1 Multivariate Equity Products	121
5.1.1 European Multivariate Equity Derivatives	122
5.1.2 Path-dependent Equity Derivatives	125
5.2 Recursions of Running Maxima and Minima	126
5.3 The Memory Feature	130
5.4 Risk-neutral Pricing Restrictions	132
5.5 Time-changed Brownian Copulas	133
5.6 Variance Swaps	135
5.7 Semi-parametric Pricing of Path-dependent Derivatives	136
5.8 The Multivariate Pricing Setting	137
5.9 H-Condition and Granger Causality	137
5.10 Multivariate Pricing Recursion	138
5.11 Hedging Multivariate Equity Derivatives	141
5.12 Correlation Swaps	144
5.13 The Term Structure of Multivariate Equity Derivatives	147
5.13.1 Altiplanos	148
5.13.2 Everest	150
5.13.3 Spread Options	150

---

<b>6 Multivariate Credit Products</b>	<b>153</b>
6.1 Credit Transfer Finance	153
6.1.1 Univariate Credit Transfer Products	154
6.1.2 Multivariate Credit Transfer Products	155
6.2 Credit Information: Equity vs CDS	158
6.3 Structural Models	160
6.3.1 Univariate Model: Credit Risk as a Put Option	160
6.3.2 Multivariate Model: Gaussian Copula	161
6.3.3 Large Portfolio Model: Vasicek Formula	163
6.4 Intensity-based Models	164
6.4.1 Univariate Model: Poisson and Cox Processes	165
6.4.2 Multivariate Model: Marshall–Olkin Copula	165
6.4.3 Homogeneous Model: Cuadras Augé Copula	167
6.5 Frailty Models	170
6.5.1 Multivariate Model: Archimedean Copulas	170
6.5.2 Large Portfolio Model: Schönbucher Formula	171
6.6 Granularity Adjustment	171
6.7 Credit Portfolio Analysis	172
6.7.1 Semi-unsupervised Cluster Analysis: <i>K-means</i>	172
6.7.2 Unsupervised Cluster Analysis: Kohonen Self-organizing Maps	174
6.7.3 (Semi-)unsupervised Cluster Analysis: Hierarchical Correlation Model	175
6.8 Dynamic Analysis of Credit Risk Portfolios	176
<b>7 Risk Capital Management</b>	<b>181</b>
7.1 A Review of Value-at-Risk and Other Measures	181
7.2 Capital Aggregation and Allocation	185
7.2.1 Aggregation: <i>C</i> -Convolution	187
7.2.2 Allocation: Level Curves	189
7.2.3 Allocation with Constraints	191
7.3 Risk Measurement of Managed Portfolios	193
7.3.1 Henriksson–Merton Model	195
7.3.2 Semi-parametric Analysis of Managed Funds	200
7.3.3 Market-neutral Investments	201
7.4 Temporal Aggregation of Risk Measures	202
7.4.1 The Square-root Formula	203
7.4.2 Temporal Aggregation by <i>C</i> -convolution	203
<b>8 Frontier Issues</b>	<b>207</b>
8.1 Lévy Copulas	207
8.2 Pareto Copulas	210
8.3 Semi-martingale Copulas	212
<b>A Elements of Probability</b>	<b>215</b>
A.1 Elements of Measure Theory	215
A.2 Integration	216
A.2.1 Expected Values and Moments	217
A.3 The Moment-generating Function or Laplace Transform	218

A.4	The Characteristic Function	219
A.5	Relevant Probability Distributions	219
A.6	Random Vectors and Multivariate Distributions	224
A.6.1	The Multivariate Normal Distribution	225
A.7	Infinite Divisibility	226
A.8	Convergence of Sequences of Random Variables	228
A.8.1	The Strong Law of Large Numbers	229
A.9	The Radon–Nikodym Derivative	229
A.10	Conditional Expectation	229
<b>B</b>	<b>Elements of Stochastic Processes Theory</b>	<b>231</b>
B.1	Stochastic Processes	231
B.1.1	Filtrations	231
B.1.2	Stopping Times	232
B.2	Martingales	233
B.3	Markov Processes	234
B.4	Lévy Processes	237
B.4.1	Subordinators	240
B.5	Semi-martingales	240
<b>References</b>		<b>245</b>
<b>Extra Reading</b>		<b>251</b>
<b>Index</b>		<b>259</b>