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Bayesian Statistics:

An Introduction

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Second Edition

A member of the Hodder Headline Group
LONDON • SYDNEY • AUCKLAND
Copublished in North, Central and South America by
John Wiley & Sons Inc., New York • Toronto

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Note: The tables in the Appendix are intended for use in conjunction with a standard set of statistical tables, for example, Lindley and Scott (1995) or Neave (1978). They extend the coverage of these tables so that they are roughly comparable with those of Isaacs *et al.* (1974) or with the tables in Appendix A of Novick and Jackson (1974). However, tables of values easily computed with a pocket calculator have been omitted. The tables have been computed using NAG routines and algorithms described in Patil (1965) and Jackson (1974).