

Discrete Dynamical Systems, Bifurcations and Chaos in Economics

WEI-BIN ZHANG COLLEGE OF ASIA PASIFIC MANAGEMENT RITSUMEIKAN ASIA PASIFIC UNIVERSITY BEPPU-SHI, OITA-KEN JAPAN



Amsterdam – Boston – Heidelberg – London – New York – Oxford Paris – San Diego – San Francisco – Singapore – Sydney – Tokyo

Contents

Pı	Preface		
C	Contents		
1	Diffe	rence equations in economics	1
	1.1	Difference equations and economic analysis	2
	1.2	Overview	7
2	Scalar linear difference equations		13
	2.1	Linear first-order difference equations	15
	2.2	Some concepts	21
	2.3	Stabilities	27
	2.4	Stabilities of nonhyperbolic equilibrium points	42
	2.5	On dissipative maps	49
	2.6	Linear difference equations of higher order	54
	2.7	Equations with constant coefficients	60
	2.8	Limiting behavior	67
3	One-	dimensional dynamical economic systems	79
	3.1	A model of inflation and unemployment	80
	3.2	The one-sector growth (OSG model)	83
	3.3	The general OSG model	88
	3.4	The overlapping-generations (OLG) model	93
	3.5	Persistence of inequality and development	97
	3.6	Growth with creative destruction	100
	3.7	Economic evolution with human capital	106
	3.8	Urbanization with human capital externalities	112
	3.9	The OSG model with money	118
	3.10	The OSG model with labor supply	126

CONTENTS

4	Time	-dependent solutions of scalar systems	135
	4.1	Periodic orbits	135
	4.2	Period-doubling bifurcations	148
	4.3	Aperiodic orbits	155
	4.4	Some types of bifurcations	163
	4.5	Liapunov numbers	170
	4.6	Chaos	177
5	Econ	omic bifurcations and chaos	185
	5.1	Business cycles with knowledge spillovers	186
	5.2	A cobweb model with adaptive adjustment	193
	5.3	Inventory model with rational expectations	195
	5.4	Economic growth with pollution	202
	5.5	The Solow and Schumpeter growth oscillations	205
	5.6	Money, growth and fluctuations	213
	5.7	Population and economic growth	219
6	High	er dimensional difference equations	227
	6.1	Phase space analysis of planar linear systems	228
	6.2	Autonomous linear difference equations	240
	6.3	Nonautonomous linear difference equations	248
	6.4	Stabilities	261
	6.5	Liapunov's direct method	270
	6.6	Linearization of difference equations	276
	6.7	Conjugacy and center manifolds	281
	6.8	The Hénon map and bifurcations	289
	6.9	The Neimark-Sacker (Hopf) bifurcations	296
	6.10	The Liapunov numbers and chaos	301
7	High	ner dimensional economic dynamics	305
	7.ĭ	An exchange rate model	306
	7.2	A two-sector OLG model	310
	7.3	Growth with government spending	315
	7.4	Growth with fertility and old age support	320
	7.5	Growth with different types of economies	327
	7.6	Unemployment, inflation and chaos	331
	7.7	Business cycles with money and capital	335
	7.8	The OSG model with heterogeneous households	341
	7.9	Path-dependent evolution with education	358
	App	endix	374
		A.7.1 Proving proposition 7.81	374
		A.7.2 Proving proposition 7.9.1	379

viii

8	8 Epilogue		385
Appendix		391	
-	A.1	Matrix theory	391
	A.2	Systems of linear equations	397
	A.3	Metric spaces	398
	A.4	The implicit function theorem	401
	A.5	The Taylor expansion and linearization	408
	A.6	Concave and quasiconcave functions	410
	A.7	Unconstrained maximization	415
	A.8	Constrained maximization	418
	A.9	Dynamical optimization	422

Bib	liog	raphy

I	nd	ex

CONTENTS