

THE THEORY OF OPEN QUANTUM SYSTEMS

Heinz-Peter Breuer and Francesco Petruccione

*Albert-Ludwigs-Universität Freiburg, Fakultät für Physik
and
Istituto Italiano per gli Studi Filosofici*

OXFORD
UNIVERSITY PRESS

CONTENTS

I PROBABILITY IN CLASSICAL AND QUANTUM PHYSICS

1	Classical probability theory and stochastic processes	3
1.1	The probability space	3
1.1.1	The σ -algebra of events	3
1.1.2	Probability measures and Kolmogorov axioms	4
1.1.3	Conditional probabilities and independence	5
1.2	Random variables	5
1.2.1	Definition of random variables	6
1.2.2	Transformation of random variables	8
1.2.3	Expectation values and characteristic function	9
1.3	Stochastic processes	11
1.3.1	Formal definition of a stochastic process	11
1.3.2	The hierarchy of joint probability distributions	12
1.4	Markov processes	14
1.4.1	The Chapman–Kolmogorov equation	14
1.4.2	Differential Chapman–Kolmogorov equation	17
1.4.3	Deterministic processes and Liouville equation	19
1.4.4	Jump processes and the master equation	21
1.4.5	Diffusion processes and Fokker–Planck equation	28
1.5	Piecewise deterministic processes	32
1.5.1	The Liouville master equation	33
1.5.2	Waiting time distribution and sample paths	33
1.5.3	Path integral representation of PDPs	37
1.5.4	Stochastic calculus for PDPs	39
1.6	Lévy processes	45
1.6.1	Translation invariant processes	46
1.6.2	The Lévy–Khintchine formula	47
1.6.3	Stable Lévy processes	51
	References	57
2	Quantum probability	59
2.1	The statistical interpretation of quantum mechanics	59
2.1.1	Self-adjoint operators and the spectral theorem	59
2.1.2	Observables and random variables	63
2.1.3	Pure states and statistical mixtures	65
2.1.4	Joint probabilities in quantum mechanics	70
2.2	Composite quantum systems	74

2.2.1	Tensor product	75
2.2.2	Schmidt decomposition and entanglement	77
2.3	Quantum entropies	79
2.3.1	Von Neumann entropy	79
2.3.2	Relative entropy	81
2.3.3	Linear entropy	82
2.4	The theory of quantum measurement	83
2.4.1	Ideal quantum measurements	83
2.4.2	Operations and effects	85
2.4.3	Representation theorem for quantum operations	87
2.4.4	Quantum measurement and entropy	92
2.4.5	Approximate measurements	93
2.4.6	Indirect quantum measurements	96
2.4.7	Quantum non-demolition measurements	102
	References	104

II DENSITY MATRIX THEORY

3	Quantum master equations	109
3.1	Closed and open quantum systems	110
3.1.1	The Liouville–von Neumann equation	110
3.1.2	Heisenberg and interaction picture	112
3.1.3	Dynamics of open systems	115
3.2	Quantum Markov processes	117
3.2.1	Quantum dynamical semigroups	117
3.2.2	The Markovian quantum master equation	119
3.2.3	The adjoint quantum master equation	124
3.2.4	Multi-time correlation functions	125
3.2.5	Irreversibility and entropy production	128
3.3	Microscopic derivations	130
3.3.1	Weak-coupling Limit	130
3.3.2	Relaxation to equilibrium	137
3.3.3	Singular-coupling limit	138
3.3.4	Low-density limit	139
3.4	The quantum optical master equation	141
3.4.1	Matter in quantized radiation fields	141
3.4.2	Decay of a two-level system	146
3.4.3	Decay into a squeezed field vacuum	149
3.4.4	More general reservoirs	152
3.4.5	Resonance fluorescence	154
3.4.6	The damped harmonic oscillator	160
3.5	Non-selective, continuous measurements	166
3.5.1	The quantum Zeno effect	166
3.5.2	Density matrix equation	167

3.6	Quantum Brownian motion	172
3.6.1	The Caldeira–Leggett model	172
3.6.2	High-temperature master equation	173
3.6.3	The exact Heisenberg equations of motion	182
3.6.4	The influence functional	192
3.7	Non-linear quantum master equations	201
3.7.1	Quantum Boltzmann equation	201
3.7.2	Mean field master equations	203
3.7.3	Mean field laser equations	205
3.7.4	Non-linear Schrödinger equation	208
3.7.5	Super-radiance	210
	References	216
4	Decoherence	219
4.1	The decoherence function	220
4.2	An exactly-solvable model	225
4.2.1	Time evolution of the total system	225
4.2.2	Decay of coherences and the decoherence factor	227
4.2.3	Coherent subspaces and system-size dependence	231
4.3	Markovian mechanisms of decoherence	232
4.3.1	The decoherence rate	233
4.3.2	Quantum Brownian motion	234
4.3.3	Internal degrees of freedom	235
4.3.4	Scattering of particles	237
4.4	The damped harmonic oscillator	242
4.4.1	Vacuum decoherence	242
4.4.2	Thermal noise	246
4.5	Electromagnetic field states	251
4.5.1	Atoms interacting with a cavity field mode	251
4.5.2	Schrödinger cat states	257
4.6	Caldeira–Leggett model	262
4.6.1	General decoherence formula	263
4.6.2	Ohmic environments	265
4.7	Decoherence and quantum measurement	269
4.7.1	Dynamical selection of a pointer basis	270
4.7.2	Dynamical model for a quantum measurement	275
	References	278
	III STOCHASTIC PROCESSES IN HILBERT SPACE	
5	Probability distributions on Hilbert space	283
5.1	The state vector as a random variable in Hilbert space	283
5.1.1	A new type of quantum mechanical ensemble	283
5.1.2	Stern–Gerlach experiment	288
5.2	Probability density functionals on Hilbert space	291

5.2.1	Probability measures on Hilbert space	291
5.2.2	Distributions on projective Hilbert space	294
5.2.3	Expectation values	297
5.3	Ensembles of mixtures	299
5.3.1	Probability density functionals on state space	299
5.3.2	Description of selective quantum measurements	301
	References	302
6	Stochastic dynamics in Hilbert space	303
6.1	Dynamical semigroups and PDPs in Hilbert space	304
6.1.1	Reduced system dynamics as a PDP	304
6.1.2	The Hilbert space path integral	311
6.1.3	Diffusion approximation	314
6.1.4	Multi-time correlation functions	316
6.2	Stochastic representation of continuous measurements	320
6.2.1	Stochastic time evolution of \mathcal{E}_P -ensembles	321
6.2.2	Short-time behaviour of the propagator	322
6.3	Direct photodetection	324
6.3.1	Derivation of the PDP	324
6.3.2	Path integral solution	330
6.4	Homodyne photodetection	335
6.4.1	Derivation of the PDP for homodyne detection	336
6.4.2	Stochastic Schrödinger equation	340
6.5	Heterodyne photodetection	342
6.5.1	Stochastic Schrödinger equation	342
6.5.2	Stochastic collapse models	345
6.6	Stochastic density matrix equations	348
6.7	Photodetection on a field mode	350
6.7.1	The photocounting formula	350
6.7.2	QND measurement of a field mode	354
	References	358
7	The stochastic simulation method	361
7.1	Numerical simulation algorithms for PDPs	362
7.1.1	Estimation of expectation values	362
7.1.2	Generation of realizations of the process	363
7.1.3	Determination of the waiting time	364
7.1.4	Selection of the jumps	367
7.2	Algorithms for stochastic Schrödinger equations	367
7.2.1	General remarks on convergence	368
7.2.2	The Euler scheme	370
7.2.3	The Heun scheme	370
7.2.4	The fourth-order Runge–Kutta scheme	370
7.2.5	A second-order weak scheme	372
7.3	Examples	373

7.3.1	The damped harmonic oscillator	373
7.3.2	The driven two-level system	377
7.4	A case study on numerical performance	380
7.4.1	Numerical efficiency and scaling laws	381
7.4.2	The damped driven Morse oscillator	383
	References	389
8	Applications to quantum optical systems	390
8.1	Continuous measurements in QED	391
8.1.1	Constructing the microscopic Hamiltonian	391
8.1.2	Determination of the QED operation	393
8.1.3	Stochastic dynamics of multipole radiation	396
8.1.4	Representation of incomplete measurements	398
8.2	Dark state resonances	401
8.2.1	Waiting time distribution and trapping state	401
8.2.2	Measurement schemes and stochastic evolution	405
8.3	Laser cooling and Lévy processes	409
8.3.1	Dynamics of the atomic wave function	410
8.3.2	Coherent population trapping	416
8.3.3	Waiting times and momentum distributions	421
8.4	Strong field interaction and the Floquet picture	428
8.4.1	Floquet theory	429
8.4.2	Stochastic dynamics in the Floquet picture	431
8.4.3	Spectral detection and the dressed atom	434
	References	437
	IV NON-MARKOVIAN QUANTUM PROCESSES	
9	Projection operator techniques	441
9.1	The Nakajima–Zwanzig projection operator technique	442
9.1.1	Projection operators	442
9.1.2	The Nakajima–Zwanzig equation	443
9.2	The time-convolutionless projection operator method	445
9.2.1	The time-local master equation	446
9.2.2	Perturbation expansion of the TCL generator	447
9.2.3	The cumulant expansion	451
9.2.4	Perturbation expansion of the inhomogeneity	452
9.2.5	Error analysis	455
9.3	Stochastic unravelling in the doubled Hilbert space	456
	References	458
10	Non-Markovian dynamics in physical systems	460
10.1	Spontaneous decay of a two-level system	461
10.1.1	Exact master equation and TCL generator	461
10.1.2	Jaynes–Cummings model on resonance	466

10.1.3 Jaynes–Cummings model with detuning	471
10.1.4 Spontaneous decay into a photonic band gap	474
10.2 The damped harmonic oscillator	474
10.2.1 The model and frequency renormalization	475
10.2.2 Factorizing initial conditions	477
10.2.3 The stationary state	481
10.2.4 Non-factorizing initial conditions	483
10.2.5 Disregarding the inhomogeneity	488
10.3 The spin-boson system	490
10.3.1 Microscopic model	490
10.3.2 Relaxation of an initially factorizing state	491
10.3.3 Equilibrium correlation functions	495
10.3.4 Transition from coherent to incoherent motion	496
References	497

V RELATIVISTIC QUANTUM PROCESSES

11 Measurements in relativistic quantum mechanics	501
11.1 The Schwinger–Tomonaga equation	502
11.1.1 States as functionals of spacelike hypersurfaces	502
11.1.2 Foliations of space-time	506
11.2 The measurement of local observables	507
11.2.1 The operation for a local measurement	508
11.2.2 Relativistic state reduction	511
11.2.3 Multivalued space-time amplitudes	514
11.2.4 The consistent hierarchy of joint probabilities	517
11.2.5 EPR correlations	521
11.2.6 Continuous measurements	523
11.3 Non-local measurements and causality	526
11.3.1 Entangled quantum probes	527
11.3.2 Non-local measurement by EPR probes	530
11.3.3 Quantum state verification	536
11.3.4 Non-local operations and the causality principle	538
11.3.5 Restrictions on the measurability of operators	544
11.3.6 QND verification of non-local states	550
11.3.7 Preparation of non-local states	554
11.3.8 Exchange measurements	555
11.4 Quantum teleportation	557
11.4.1 Coherent transfer of quantum states	557
11.4.2 Teleportation and Bell-state measurement	560
11.4.3 Experimental realization	562
References	565
12 Open quantum electrodynamics	568
12.1 Density matrix theory for QED	569

12.1.1	Field equations and correlation functions	569
12.1.2	The reduced density matrix	576
12.2	The influence functional of QED	577
12.2.1	Elimination of the radiation degrees of freedom	577
12.2.2	Vacuum-to-vacuum amplitude	583
12.2.3	Second-order equation of motion	585
12.3	Decoherence by emission of bremsstrahlung	588
12.3.1	Introducing the decoherence functional	589
12.3.2	Physical interpretation	593
12.3.3	Evaluation of the decoherence functional	596
12.3.4	Path integral approach	607
12.4	Decoherence of many-particle states	614
	References	617
	Index	619