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# MODERN PROBABILITY AND STATISTICS

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## Generalized Poisson Models and their Applications in Insurance and Finance

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