

Second Edition

Robert A. Adams and John J. F. Fournier

Department of Mathematics
The University of British Columbia
Vancouver, Canada



Amsterdam Boston Heidelberg London New York Oxford Paris San Diego San Francisco Singapore Sydney Tokyo

CONTENTS

	Preface	ix
	List of Spaces and Norms	xii
1.	PRELIMINARIES	1
	Notation	1
	Topological Vector Spaces	3
	Normed Spaces	4
	Spaces of Continuous Functions	10
	The Lebesgue Measure in \mathbb{R}^n	13
	The Lebesgue Integral	16
	Distributions and Weak Derivatives	. 19
2.	THE LEBESGUE SPACES $L^p(\varOmega)$	23
	Definition and Basic Properties	23
	Completeness of $L^p(\Omega)$	29
	Approximation by Continuous Functions	31
	Convolutions and Young's Theorem	32
	Mollifiers and Approximation by Smooth Functions	36
	Precompact Sets in $L^p(\Omega)$	38
	Uniform Convexity	41
	The Normed Dual of $L^p(\Omega)$	45
	Mixed-Norm L^p Spaces	49
	The Marcinkiewicz Interpolation Theorem	52.

ví Contents

3.	THE SOBOLEV SPACES $W^{m,p}(\varOmega)$	59
	Definitions and Basic Properties	59
	Duality and the Spaces $W^{-m,p'}(\Omega)$	62
	Approximation by Smooth Functions on Ω	65
	Approximation by Smooth Functions on \mathbb{R}^n	67
	Approximation by Functions in $C_0^{\infty}(\Omega)$	70
	Coordinate Transformations	77
4.	THE SOBOLEV IMBEDDING THEOREM	79
	Geometric Properties of Domains	81
	Imbeddings by Potential Arguments	87
	Imbeddings by Averaging	93
	Imbeddings into Lipschitz Spaces	99
	Sobolev's Inequality	101
	Variations of Sobolev's Inequality	104
	$W^{m,p}(\Omega)$ as a Banach Algebra	106
	Optimality of the Imbedding Theorem	108
	Nonimbedding Theorems for Irregular Domains	111
	Imbedding Theorems for Domains with Cusps	115
	Imbedding Inequalities Involving Weighted Norms	119
	Proofs of Theorems 4.51–4.53	131
5.	INTERPOLATION, EXTENSION, AND APPROXIMATION	135
	THEOREMS	
	Interpolation on Order of Smoothness	135
	Interpolation on Degree of Sumability	139
	Interpolation Involving Compact Subdomains	143
	Extension Theorems	146
	An Approximation Theorem	159
	Boundary Traces	163
6.	COMPACT IMBEDDINGS OF SOBOLEV SPACES	167
	The Rellich-Kondrachov Theorem	167
	Two Counterexamples	173
,	Unbounded Domains — Compact Imbeddings of $W_0^{m,p}(\Omega)$	175
	An Equivalent Norm for $W_0^{m,\hat{p}}(\Omega)$	183
	Unbounded Domains — Decay at Infinity	186
	Unbounded Domains — Compact Imbeddings of $W^{m,p}(\Omega)$	195
	Hilbert-Schmidt Imbeddings	200

7.	FRACTIONAL ORDER SPACES	205
	Introduction	205
	The Bochner Integral	206
	Intermediate Spaces and Interpolation — The Real Method	208
	The Lorentz Spaces	221
	Besov Spaces	228
	Generalized Spaces of Hölder Continuous Functions	232
	Characterization of Traces	234
	Direct Characterizations of Besov Spaces	241
	Other Scales of Intermediate Spaces	247
	Wavelet Characterizations	256
8.	ORLICZ SPACES AND ORLICZ-SOBOLEV SPACES	261
	Introduction	261
	N-Functions	262
	Orlicz Spaces	266
	Duality in Orlicz Spaces	272
	Separability and Compactness Theorems	274
	A Limiting Case of the Sobolev Imbedding Theorem	277
•	Orlicz-Sobolev Spaces	281
	Imbedding Theorems for Orlicz-Sobolev Spaces	282
	References	295
	Index	301