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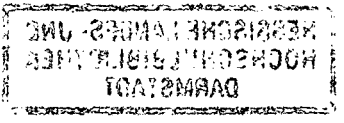
Piecewise Linear Control Systems

A Computational Approach

With 69 Figures



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