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# Spectral Analysis and Filter Theory in Applied Geophysics

With 164 Figures and 23 Tables



Springer

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