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The Measurement of Market Risk

Modelling of Risk Factors, Asset Pricing,
and Approximation of Portfolio Distributions



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Contents

1. Introduction	1
1.1 The Need for Risk Measurement	1
1.2 The Nature of Financial Risk	3
1.3 Formal Framework	5
1.3.1 Modelling the Uncertainty	6
1.3.2 The Information Structure	8
1.4 Problem Statement	9
1.5 Structure of the Book	11
1.6 Test Environment	16
1.6.1 Environment I	17
1.6.2 Environment II	18
2. Risk and Risk Measures	21
2.1 The Investment Decision	22
2.1.1 Utility Theory and Expected Utility Hypothesis	23
2.1.2 Rules for the Ordering of Uncertain Prospects	33
2.2 The Capital Requirement Decision	41
2.2.1 Value-at-Risk	42
2.2.2 Coherent Risk Measures	44
2.3 Summary	47
3. Modelling the Dynamics of the Risk Factors	49
3.1 Statistical Definitions	50
3.1.1 Stochastic Processes: Basic Definitions	50
3.1.2 Properties of Stochastic Processes	52
3.1.3. Basic Stochastic Processes	53
3.2 The Economic Assumption: the Efficient Market Hypothesis	59
3.3 Empirical Evidence for the Returns	62
3.3.1 Calendar Effects	62
3.3.2 Leptokurtosis and Weak Evidence of Skewness	62
3.3.3 The Autocorrelation of the Squared Returns	64
3.4 Models for the Risk Factor Dynamics	66
3.4.1 The Generic Model for the Log-returns	67
3.4.2 ARCH Models	68

3.4.3	Stochastic Variance Models	71
3.5	Empirical Analysis of the Returns on Swiss Stocks	80
3.5.1	The Data	80
3.5.2	Descriptive Statistics and Correlation	80
3.5.3	Implementation of an Alternative Model	82
3.5.4	Impact of the Alternative Modelling	90
3.6	Continuous-Time Models	92
3.7	Summary	97
4.	Valuation of Financial Instruments	99
4.1	Principles of Valuation	100
4.1.1	Valuation by Arbitrage	102
4.2	Cash Instruments	116
4.2.1	Equities	116
4.2.2	Fixed-Income Instruments	117
4.3	Futures and Forwards	119
4.4	Options	121
4.4.1	The Black-Scholes Analysis	122
4.4.2	Risk-Neutral Valuation	124
4.4.3	Numerical Approaches	126
4.5	Approximation of the Value Function	133
4.5.1	Global Taylor Approximation for Option Pricing	135
4.5.2	Piecewise Taylor Approximations	136
5.	Approximation of the Portfolio Distribution	141
5.1	Analytical Methods	142
5.1.1	Delta Approximation	144
5.1.2	Delta-Gamma Approximation	146
5.2	Generation of Scenarios	151
5.2.1	The Pseudo-Random Method	152
5.2.2	The Quasi-Random Method	153
5.2.3	Generation of Distributions for the Risk Factors	159
5.3	Monte Carlo Simulation	165
5.3.1	Error Analysis	166
5.3.2	Variance Reduction Techniques	173
5.4	TheBDPQA	177
5.4.1	Simplices	178
5.4.2	• Simplicial Coverage of the Risk Factor Distribution ...	180
5.4.3	Barycentric Discretisation	183
5.4.4	Approximation of the Portfolio Distribution	185
5.4.5	Refinement Strategies	188
5.4.6	Numerical Example	191
5.5	Benchmarking the BDPQA	195
5.5.1	The Choice of the Holding Period	201
5.6	Summary	202

6. Sample Estimation of Risk Measures	205
6.1 Introduction	205
6.2 Order Statistics	206
6.2.1 Distribution of Order Statistics	206
6.2.2 Moments of Order Statistics	209
6.2.3 Confidence Interval for Population Quantiles	210
6.3 Quantile Estimators Based on Order Statistics	213
6.3.1 Linear Combination of Several Order Statistics	214
6.4 Kernel-Based Estimators	215
6.4.1 Accuracy of the Estimate Density	217
6.4.2 Bandwidth Selection	221
6.4.3 Quantile Estimation Based on the Kernel Density Method	223
6.5 Comparison of the Quantile Estimators	224
6.6 Summary	227
7. Conclusion and Outlook	229
7.1 Summary'	229
7.2 The Issue of Credit Risk	232
7.3 Outlook	233
A. Probability and Statistics	235
A.1 Probabilistic Modelling	235
A.2 Random Variable	237
A.2.1 Distribution Function	238
A.2.2 Moments	239
A.2.3 Independence and Correlation	241
A.2.4 Conditional Probability and Expectation	242
A.2.5 Stochastic Processes and Information Structure	243
A.2.6 Martingales	244
A.3 Selected Distributions	245
A.3.1 Basic Distributions	245
A.3.2 Elliptically Contoured Distributions	246
A.3.3 Stable Distribution	248
A.4 Types of Convergence	249
A.5 Sampling Theory	251
Bibliography	253
List of Figures	265
List of Tables	267
Index	269