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# Power Systems Harmonics

Fundamentals, Analysis  
and Filter Design

With 263 Figures



Springer

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