COOPERATIVE GAMES, SOLUTIONS AND APPLICATIONS

by

THEO DRIESSEN
Assistant Professor, Faculty of Applied Mathematics, University of Twente, The Netherlands

KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

TABLE OF CONTENTS

PREFACE v

NOTATION xiii

CHAPTER I
COOPERATIVE GAMES AND EXAMPLES
1. The Tennessee Valley Authority 1
2. Cooperative games in characteristic function form 3
3. A production economy with landowners and peasants 4
4. An exchange economy with traders of two types 5
5. The airport game 6
6. The bankruptcy game 7
7. Cooperative water resource development in Japan 8
8. Simple games 9
9. Notions 11

CHAPTER II
SOLUTION CONCEPTS FOR COOPERATIVE GAMES AND RELATED SUBJECTS
1. Notions 13
2. The Shapley value 15
3. The stable sets 19
4. The core and the strong ϵ-cores 20
5. The bargaining set $\mathcal{M}$ 24
6. The kernel and the prekernel 26
7. The nucleolus 37
8. Balancedness 47

CHAPTER III
THE τ-VALUE
1. The upper vector, the concession vector and the gap function of a game 57
2. The τ-value of a quasibalanced game 59
3. Necessary and sufficient conditions for the τ-value on QB^n to belong to the core 62
4. An axiomatic characterization of the τ-value on QB^n 70
5. 1-Convex games 73
6. Semiconvex games 76
7. The τ-value of a quasibalanced simple game 79
8. The τ-value of a game with a nonempty imputation set 81

CHAPTER IV
A GAME THEORETIC APPROACH TO THE COST ALLOCATION PROBLEM BY MEANS OF THE τ-VALUE, THE NUCLEOLUS AND THE SHAPLEY VALUE
1. The TVA cost allocation problem 91
2. The τ-value in comparison with cost allocation methods based on separable and nonseparable costs 92
3. The nucleolus in comparison with the egalitarian nonseparable cost method 98
4. The airport cost allocation problem 104

CHAPTER V
CONVEX GAMES AND SOLUTION CONCEPTS
1. Convex games 111
2. Examples of convex games 116
3. The core of a convex game 120
4. The Shapley value and the τ-value of a convex game 129
5. The stable set of a convex game 132
6. The bargaining set $\mathcal{M}$ of a convex game 135
7. The kernel and the prekernel of a convex game 138

CHAPTER VI
DIVISION RULES AND ASSOCIATED GAME THEORETIC SOLUTIONS FOR BANKRUPTCY PROBLEMS
1. Introduction 145
2. The recursive completion division rule in comparison with the Shapley value 146
3. The contested garment consistent division rule in comparison with the nucleolus 151
4. The adjusted proportional division rule in comparison with the τ-value 159
5. Axiomatic characterizations of division rules in terms of self-duality 162

CHAPTER VII
k-CONVEX GAMES AND SOLUTION CONCEPTS
1. k-Convex games 171
2. Characterizations of k-convexity in terms of the gap function 178
3. Examples of k-convex games 184
4. The core of a k-convex game 194
5. The τ-value of a k-convex game 200
6. The Shapley value of a k-convex game 203

7. The kernel and the prekernel of a k-convex game 206
8. The bargaining set $\mathcal{M}$ of a k-convex game 208

REFERENCES 211

AUTHOR INDEX 219

SUBJECT INDEX 221