Selected Works of A. N. Kolmogorov

Volume I Mathematics and Mechanics

edited by

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 $\mathbf{x}_{1}^{*} = \mathbf{x}_{1}^{*} + \mathbf{x}_{2}^{*}$

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