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Cost-Minimizing Choice Behavior in Transportation Planning

A Theoretical Framework for Logit Models

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