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Complex and Chaotic Nonlinear Dynamics-

Advances in Economics and Finance, Mathematics and Statistics



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