## Population Ageing and Economic Growth

Education Policy and Family Policy in a Model of Endogenous Growth

With 7 Figures and 32 Tables



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