

Cho-Ho Chu

# Matrix Convolution Operators on Groups

$$\begin{aligned} & \text{Definition: } \mathcal{F}(G) = \{f: G \rightarrow \mathbb{C}\} \\ & \text{Convolution: } (f * g)(x) = \int_G f(y)g(y^{-1}x)dy \\ & \text{Group: } G = \{e, a, b, c\} \\ & \text{Matrix: } M = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix} \end{aligned}$$

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