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Information Geometry

Near Randomness
and Near Independence

 Springer

Volume 1
Information Geometry
of Probability
Distributions

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