

Anadijiban Das

The Special Theory of Relativity

A Mathematical Exposition

With 27 Illustrations



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