

Juraj Hromkovič

# Communication Complexity and Parallel Computing

With 40 Figures.



Springer

# Table of Contents

1	Introduction . . . . .	1
1.1	Motivation and Aims . . . . .	1
1.2	Concept and Organization . . . . .	4
1.3	How to Read the Book . . . . .	6
2	Communication Protocol Models . . . . .	7
2.1	Basic Notions . . . . .	7
2.1.1	Introduction . . . . .	7
2.1.2	Alphabets, Words, and Languages . . . . .	7
2.1.3	Boolean Functions and Boolean Matrices . . . . .	11
2.1.4	Representation of Computing Problems . . . . .	16
2.1.5	Exercises . . . . .	20
2.2	Communication Complexity According to a Fixed Partition . . . . .	23
2.2.1	Definitions . . . . .	23
2.2.2	Methods for Proving Lower Bounds . . . . .	30
2.2.3	Theoretical Properties of Communication Complexity According to a Fixed Partition . . . . .	53
2.2.4	Exercises . . . . .	57
2.2.5	Research Problems . . . . .	59
2.3	Communication Complexity . . . . .	60
2.3.1	Introduction . . . . .	60
2.3.2	Definitions . . . . .	61
2.3.3	Lower Bound Methods . . . . .	62
2.3.4	Theoretical Properties of Communication Complexity . . . . .	70
2.3.5	Communication Complexity and Chomsky Hierarchy . . . . .	77
2.3.6	Exercises . . . . .	82
2.3.7	Research Problems . . . . .	83
2.4	One-Way Communication Complexity . . . . .	83
2.4.1	Introduction . . . . .	83
2.4.2	Definitions . . . . .	84
2.4.3	Methods for Proving Lower Bounds . . . . .	86
2.4.4	Communication Complexity Versus One-way Communication Complexity . . . . .	92
2.4.5	Exercises . . . . .	95
2.4.6	Research Problems . . . . .	96

2.5	Nondeterministic Communication Complexity and Randomized Protocols . . . . .	97
2.5.1	Introduction . . . . .	97
2.5.2	Nondeterministic Protocols . . . . .	98
2.5.3	Lower Bounds on Nondeterministic Communication Complexity . . . . .	105
2.5.4	Deterministic Protocols Versus Nondeterministic Protocols . . . . .	109
2.5.5	Randomized Protocols . . . . .	115
2.5.6	Randomness Versus Nondeterminism and Determinism . . . . .	123
2.5.7	Exercises . . . . .	127
2.5.8	Research problems . . . . .	130
2.6	An Improved Model of Communication Protocols . . . . .	131
2.6.1	Introduction . . . . .	131
2.6.2	Definitions . . . . .	132
2.6.3	Lower Bound Methods . . . . .	135
2.6.4	Communication Complexity Versus $s$ -communication Complexity . . . . .	139
2.6.5	Some Properties of $s$ -communication Complexity . . . . .	140
2.6.6	Exercises . . . . .	143
2.6.7	Problems . . . . .	144
2.7	Bibliographical Remarks . . . . .	144
3	Boolean Circuits . . . . .	151
3.1	Introduction . . . . .	151
3.2	Definitions and Fundamental Properties . . . . .	152
3.2.1	Introduction . . . . .	152
3.2.2	Boolean Circuit Models . . . . .	152
3.2.3	Fundamental Observations . . . . .	159
3.2.4	Exercises . . . . .	163
3.3	Lower Bounds on the Area of Boolean Circuits . . . . .	164
3.3.1	Introduction . . . . .	164
3.3.2	Definitions . . . . .	164
3.3.3	Lower Bounds on the Area Complexity Measures . . . . .	167
3.3.4	A Comparison of two Area Complexity Measures . . . . .	173
3.3.5	Three-Dimensional Layout . . . . .	179
3.3.6	Exercises . . . . .	182
3.3.7	Problems . . . . .	184
3.4	Topology of Circuits and Lower Bounds . . . . .	185
3.4.1	Introduction . . . . .	185
3.4.2	Separators . . . . .	185
3.4.3	Lower Bounds on Boolean Circuits with a Sublinear Separator . . . . .	192

3.4.4	Circuit Structures for Which Communication Complexity Does Not Help . . . . .	196
3.4.5	Planar Boolean Circuits . . . . .	200
3.4.6	Exercises . . . . .	215
3.4.7	Problems . . . . .	217
3.5	Lower Bounds on the Size of Unbounded Fan-in Circuits . . . . .	217
3.5.1	Introduction . . . . .	217
3.5.2	Method of Communication Complexity of Infinite Bases . . . . .	218
3.5.3	Unbounded Fan-in Circuits with Sublinear Vertex-Separators . . . . .	222
3.5.4	Exercises . . . . .	224
3.5.5	Problems . . . . .	225
3.6	Lower Bounds on the Depth of Boolean Circuits . . . . .	225
3.6.1	Introduction . . . . .	225
3.6.2	Monotone Boolean Circuits . . . . .	226
3.6.3	Communication Complexity of Relations . . . . .	229
3.6.4	Characterizations of Circuit Depth by the Communication Complexity of Relations . . . . .	231
3.6.5	Exercises . . . . .	236
3.6.6	Research Problems . . . . .	237
3.7	Bibliographical Remarks . . . . .	237
4	VLSI Circuits and Interconnection Networks . . . . .	241
4.1	Introduction . . . . .	241
4.2	Definitions . . . . .	242
4.2.1	Introduction . . . . .	242
4.2.2	A VLSI circuit Model . . . . .	242
4.2.3	Complexity Measures . . . . .	247
4.2.4	Probabilistic Models . . . . .	250
4.2.5	Exercises . . . . .	251
4.3	Lower Bounds on VLSI Complexity Measures . . . . .	252
4.3.1	Introduction . . . . .	252
4.3.2	Lower Bounds on Area Complexity . . . . .	252
4.3.3	Lower Bounds on Tradeoffs of Area and Time . . . . .	254
4.3.4	VLSI circuits with Special Communication Structures . . . . .	258
4.3.5	Exercises . . . . .	263
4.3.6	Problems . . . . .	264
4.4	Interconnection Networks . . . . .	264
4.4.1	Introduction . . . . .	264
4.4.2	A Model of Interconnection Networks . . . . .	265
4.4.3	Separators and Lower Bounds . . . . .	266
4.4.4	Exercises . . . . .	270
4.4.5	Problems . . . . .	270

4.5	Multilective VLSI circuits . . . . .	270
4.5.1	Introduction and Definitions . . . . .	270
4.5.2	Multilectivity Versus Semilectivity . . . . .	271
4.5.3	Lower Bounds on Multilective VLSI programs . . . . .	272
4.5.4	Exercises . . . . .	279
4.5.5	Problems . . . . .	280
4.6	Bibliographical Remarks . . . . .	280
5	Sequential Computations . . . . .	283
5.1	Introduction . . . . .	283
5.2	Finite Automata . . . . .	284
5.2.1	Introduction . . . . .	284
5.2.2	Definitions . . . . .	285
5.2.3	One-Way Communication Complexity and Lower Bounds on the Size of Finite Automata . . . . .	287
5.2.4	Uniform Protocols . . . . .	288
5.2.5	Exercises . . . . .	294
5.2.6	Research Problems . . . . .	295
5.3	Turing Machines . . . . .	295
5.3.1	Introduction . . . . .	295
5.3.2	Time Complexity of Classical Turing Machines . . . . .	296
5.3.3	Sequential Space and Time-Space Complexity . . . . .	299
5.3.4	Exercises . . . . .	301
5.3.5	Research Problems . . . . .	302
5.4	Decision Trees and Branching Programs . . . . .	302
5.4.1	Introduction . . . . .	302
5.4.2	Definitions . . . . .	303
5.4.3	Capacity of Branching Programs . . . . .	306
5.4.4	Lower Bounds on the Depth of Decision Trees . . . . .	309
5.4.5	Exercises . . . . .	310
5.4.6	Research Problems . . . . .	311
5.5	Bibliographical Remarks . . . . .	311
	References . . . . .	317
	Index . . . . .	331