

**INTRODUCTION TO THE THEORY
AND APPLICATION OF DATA
ENVELOPMENT ANALYSIS**

A Foundation Text with Integrated Software

by

EMMANUEL THANASSOULIS

Aston University, Birmingham, United Kingdom.

Contents

List of Tables		xiii
List of Figures		*v
Preface		xvii
Abbreviations		xxii
1-	INTRODUCTION TO PERFORMANCE MEASUREMENT	1
1.1	Introduction	1
1.2	Why Measure Performance?	1
1.3	Performance Measurement Methods in Outline	4
	1.3.1 Unit of assessment	4
	1.3.2 Performance Indicators	5
	1.3.3 Modelling Methods of Comparative Performance Measurement	6
	1.3.3.1 Parametric methods for measuring comparative performance	6
	1.3.3.2 Non-Parametric Methods for Measuring Comparative Performance	9
1.4	Some Areas Where Uses of DEA Have Been Reported	13
	1.4.1 Financial Services	13
	1.4.2 Regulation	15
	1.4.3 Police Services	17
1.5	Conclusion	19
2.	DEFINITIONS OF EFFICIENCY AND RELATED MEASURES	21
2.1	Introduction	21
2.2	Unit of Assessment and Input-Output variables	21
2.3	Pareto-efficiency and Measures of Input and Output Efficiency.	22

2.4	Input Overall, Allocative and Technical Efficiencies	26
2.5	An Illustration	29
2.6	Questions	31
	Appendix 2.1: Mathematical Definitions	32
	Appendix 2.2: Deriving Graphical Measures of Input Efficiencies	34
3.	DATA ENVELOPMENT ANALYSIS UNDER CONSTANT RETURNS TO SCALE: BASIC PRINCIPLES	37
3.1	Introduction	37
3.2	Basic Steps in Measuring Efficiency by DEA	37
	3.2.1 Constructing a Production Possibility Set in the Single-input Single-output Case	38
	3.2.2 Using the PPS to Derive Efficiency Measures	40
3.3	Using Linear Programming to Measure Efficiency in the Single-input Single-output Case	41
3.4	Using DEA To Measure Technical Input Efficiency in the Single-Output Multi-input Case: A Graphical Illustration	45
3.5	Using Linear Programming to Measure Technical input Efficiency in the Single-output Multi-input Case	49
3.6	Using DEA to Measure Technical Output Efficiency in the Single-input Multi-output Case: A Graphical Illustration	52
3.7	Using Linear Programming to Measure Technical Output Efficiency in the Single-input Multi-Output Case	54
3.8	Questions	57
	Appendix 3.1: Introduction to Linear Programming	59
	Appendix 3.2: Postulates for Constructing the Production Possibility Set in DEA Under Constant Returns to Scale	64
4.	DATA ENVELOPMENT ANALYSIS UNDER CONSTANT RETURNS TO SCALE: GENERAL MODELS	65
4.1	Introduction	65
4.2	A General Purpose Linear Programming Model for Assessing Technical Input Efficiency	65
4.3	A General Purpose Linear Programming Model for Assessing Technical Output Efficiency	68
4.4	Value-Based DEA Models	71
4.5	Interpretation of Value-Based DEA Models	74

4.6	Efficient Peers and Targets in DEA	78
4.6.1	Targets	78
4.6.2	Efficient Peers	80
4.7	Input and Output Allocative Efficiencies	81
4.7.1	Cost Minimising Efficiencies	81
4.7.2	Revenue Maximising Efficiencies	83
4.8	Questions	85
5.	USING DATA ENVELOPMENT ANALYSIS IN PRACTICE	89
5.1	Introduction	89
5.2	Choosing Inputs and Outputs in A DEA Assessment	89
5.3	Information Obtained in the Course of a DEA Assessment	92
5.4	Interpreting the Solution of a DEA Envelopment Model	93
5.4.1	Pareto-Inefficient DMUs	94
5.4.2	Pareto-Efficient DMUs	97
5.5	Interpreting the Solution of a Value-based Dea Model	98
5.5.1	Pareto-Efficient DMUs	99
5.5.2	Pareto-Inefficient DMUs	103
5.6	An Illustrative Use of <i>Warwick DEA Software</i>	105
5.7	Practical Tips for Carrying Out DEA Assessments	111
5.8	Questions	114
	Appendix 5.1: Warwick DEA Software	19
	Appendix 5.2	120
6.	DATA ENVELOPMENT ANALYSIS UNDER VARIABLE RETURNS TO SCALE	123
6.1	Introduction	123
6.2	The Concept of Returns to Scale	124
6.3	Assessing DEA Efficiency Under Variable Returns to Scale: A Graphical Illustration	125
6.4	Assessing DEA Efficiency Under Variable Returns to Scale: The Generic Envelopment Model	129
6.5	Value-based DEA Models Under VRS	136
6.6	Scale Efficiency, Returns to Scale and Most Productive Scale Size	139
6.6.1	Scale Efficiency	140
6.6.2	Identifying Returns to Scale By Means of DEA Models	142
6.6.3	Most Productive Scale Size	147
6.7	Practical Use of DEA Models Under VRS	153

	6.7.1 Envelopment Models	153
	6.7.2 Value-based Models	
6.8	DEA Assessments Under VRS By Means of <i>Warwick DEA SoftM>are</i>	
6.9	DEA Assessments Under Non-Increasing or Npn- Decreasing Returns to Scale	'58
6.10	Questions	160
7.	ASSESSING POLICY EFFECTIVENESS AND PRODUCTIVITY CHANGE USING DEA	163
7.1	Introduction	163
7.2	Disentangling Managerial And Policy Efficiency: An outline	164
7.3	Disentangling Managerial and Policy Efficiency: The generic approach	168
7.4	Disentangling Managerial and Policy Efficiency: Illustrative examples	171
7.5	Assessing Productivity Change By Means of DEA: A graphical introduction to the Malmquist Index	175
7.6	Assessing Productivity Change By Means of DEA: The Malmquist Index in the general case	181
7.7	Assessing Productivity Change Using Malmquist Indices: Illustrative example	184
7.8	Capturing the Impact of Scale Size Changes on Productivity	189
7.9	The Cost Malmquist Type Index	
7.10	Questions	
	Appendix 7.1	
8.	INCORPORATING VALUE JUDGEMENTS IN DEA ASSESSMENTS	199
8.1	Introduction	199
8.2	Why we May Want to Incorporate Value Judgements in DEA Assessments	199
8.3	Methods for Incorporating Value Judgements in DEA	201
8.4	Using Weights Restrictions to Incorporate Value Judgements in DEA	201
	8.4.1 Restrictions Applied to DEA Weights	202
	8.4.2 Restrictions Applied to Virtual Inputs and Outputs	203
8.5	Some Approaches to Estimating the Parameters of Weights Restrictions	204

8.6	Interpreting the Results of DEA Models with Weights Restrictions	209
8.6.1	Effects of Weights Restrictions on the Interpretation of the Efficiency Measure Yielded By a DEA Model	210
8.6.2	Effects of Weights Restrictions on DEA Targets	211
8.6.3	Additional Effects of Absolute Weights Restrictions	212
8.7	Using unobserved DMUs to incorporate value Judgements in DEA	212
8.7.1	A Procedure for Improving Envelopment in DEA	214
8.7.2	Advantages and Drawbacks of UDMUs in Incorporating Value Judgements in DEA.	217
8.8	Questions	219
	Appendix 8.1: Some Features of the DEA Efficiency Measure Under Weights Restrictions	222
	Appendix 8.2: Some Features of Targets Yielded by Dea Models Under Weights Restrictions	225
9.	EXTENSIONS TO BASIC DEA MODELS	227
9.1	Introduction	227
9.2	Assessing Efficiency Under Exogenously Fixed Input-Output Variables	227
9.3	Identifying Preferred Pareto-Efficient Input-Output Levels by DEA	235
9.3.1	Case Where no Input or Output May Deteriorate	236
9.3.2	Case Where Some Input or Output May Deteriorate	238
9.4	Assessing Dmus in the Presence of Categorical Variables	244
9.5	Questions	248
10.	A LIMITED USER GUIDE FOR <i>WARWICK DEA SOFTWARE</i>	251
10.1	Introduction	251
10.2	Preparing your Data Input before you Invoke the Programme	252
10.3	Initiating a Run of the Programme	253
10.4	Options Menu	253
10.5	The Run Menu	258
10.6	The Advanced Menu	265
10.7	File Flandling	267
10.8	Some Common Error Messages	268

Author Index	271;
Topic Index	273
References	277