

Regression Analysis and Linear Models

Concepts, Applications, and Implementation

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Data files for the examples used in the book and files containing the SPSS and SAS versions of RLM are available on the companion web page at www.afhayes.com.